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S. & C.2667 STABILITY AND CONTROL SUB-COMMITTEE S. & C.2667

AERONAUTICAL RESEARCH COUNCIL

# Note on the Longitudinal Stability of Supersonic Aircraft and Missiles

- By -
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## Summary

This note deals with a quite crude calculation of the longitudinal stability of a hypothetical supersonic design, but nevertheless appears to indicate quite clearly what the general nature of the stability of supersonic aircraft will be. It is shown that the longitudinal biquadratic splits very sharply into two quadratics and that the characteristics of the longitudinal disturbed motion are a well-damped oscillation of quite short period and a phugoid oscillation of nearly zero damping and very long period. The periods and damping factors can be roughly estimated by very simple means. It is not claimed that these results are in any way novel, but the writer felt that they might not have been generally appreciated by those not directly working on the subject of supersonic stability.

The plan of the hypothetical aeroplane is given in Fig.1 and was arrived at by postulating that it should fly at $M = 1.8$ at 40,000 ft. at a lift coefficient of 0.1. As will be seen in the sequel, very wide variations in these assumptions would not affect the basic conclusions drawn for the analysis. The following leading dimensions and quantities come from the above assumptions and from the sketch plan, which was merely completed by eye. The C.G. was taken to be at half length.

| | |
|---|---|
| Overall length | 50 ft. |
| Wing area (net) | 195 sq. ft. |
| Tail area (net) | 30 sq. ft. |
| Tail leverage | 20 ft. |
| Wing CP behind C.G. | 3 ft. |
| Sweep | about 45° |
| Weight | 18,000 lb (560 slugs mass) |
| Radius of gyration | 12 ft. |

$$\left.\begin{aligned} q &= \tfrac{1}{2}\rho V^2 = 920 \\ qS &= 180,000 \end{aligned}\right\} \text{ at } M = 1.8.$$

The lift was assumed to be all on the wings and the values of $dC_L/d\alpha$ (used for both wings and tail) was obtained from a generalised curve of lift slope at supersonic speeds, obtained from various swept-back and delta wing tests. The following basic table was derived:-

M/

| M | U (ft/sec.) | $dC_L/d\alpha$ | $dC_L/d\alpha^\circ$ |
|---|---|---|---|
| 1.3 | 1260 | 3.38 | 0.0591 |
| 1.5 | 1450 | 2.60 | 0.0455 |
| 1.7 | 1640 | 2.25 | 0.0392 |
| 1.9 | 1840 | 2.00 | 0.0348 |

The old notation (of Bairstow) was used with X axis along wind and the derivatives were calculated as follows:-

$$\underline{Z_w} = \frac{dZ}{dw} = \frac{1}{U}\frac{dZ}{d\alpha} = -\frac{1}{U} qS \frac{dC_L}{d\alpha}$$

$$\underline{Z_u} = \frac{dZ}{dU} = \frac{2Z}{U} = -\frac{2W}{U}$$

$\underline{X_u}$

The angle of incidence only varies from 3.26 to 2.57 degrees over the range of M*. It was assumed that this angle would be close to that for maximum L/D and that this quantity would be of order 5.

Hence $X_u = Z_u/5$.

$\underline{M_w}$

From American wind tunnel tests on a body very similar to that assumed it was found that $dC_m/d\alpha^\circ$ was 0.00184 expressed on wing area and with the length of the body as the characteristic dimension, and that it was very nearly independent of M. Interference between wing and tail was neglected.

$$\text{Hence } \frac{dC_m}{d\alpha^\circ} = 0.00184 - \frac{3}{50}\frac{dC_L}{d\alpha^\circ} - \frac{20}{50}\frac{dC_L}{d\alpha^\circ}\frac{S_T}{S_W}$$

$$= 0.00184 - 0.0122\, dC_L/d\alpha^\circ.$$

In these moment slopes $\alpha$ was in degrees.

$$\text{Therefore } \frac{dM}{dw} = \frac{1}{U} qSl \frac{dC_m}{d\alpha^\circ} \times 57.3$$

where $l$ is the body length.

$\underline{M_q}$

This was taken as wholly due to the tail and therefore

$$\frac{dM}{dq} = -\frac{l^2}{U}\frac{dC_L}{d\alpha} S_T \tfrac{1}{2}\rho V^2$$

where $l$ is here the tail leverage.

All/

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*This seems to be a general characteristic of supersonic flight and occurs because the lift slope is inversely proportional to $\sqrt{M^2-1}$ and $\frac{M^2}{\sqrt{M^2-1}}$ varies little from M = 1.2 to M = 2.

All other derivatives were assumed negligible.

The table below gives the values obtained; for convenience the mass and moment of inertia have been included in the derivative, e.g., the tabulated $Z_u$ is $Z_u/m$, $M_q$ is $M_q/B$, etc.

| M | $-X_u$ | $-Z_u$ | $-Z_w$ | $-M_w$ | $-M_q$ |
|---|---|---|---|---|---|
| 1.3 | 0.0102 | 0.0510 | [illegible] | [illegible] | 0.191 |
| 1.5 | 0.0088 | [illegible] | 0.394 | 0.0113 | 0.169 |
| 1.7 | 0.0076 | 0.0392 | 0.394 | 0.0102 | 0.166 |
| 1.9 | 0.0070 | 0.0350 | 0.381 | 0.0093 | 0.166 |

The coefficients of the biquadratic with only those five derivatives are:-

$$A = -X_u - Z_w - M_q$$

$$B = Z_w M_q - U M_w + X_u (M_q + Z_w)$$

$$C = -X_u (Z_w M_q - U M_w)$$

$$D = U Z_u M_w .$$

These are set out in detail below for the case of M = 1.5 so as to exhibit the relative values of the various terms.

<u>**M = 1.5**</u>

$$A = 0.0088 + 0.394 + 0.169 = 0.572$$

$$B = [illegible] + 16.5 + 0.0088 \times 0.563 = 16.6$$

$$C = [illegible] \times 16.6 = 0.146$$

$$D = [illegible] \times [illegible] \times 0.0113 = 0.0161 .$$

The biquadratic is:-

$$\lambda^4 + 0.572\,\lambda^3 + 16.6\,\lambda^2 + 0.146\,\lambda + 0.0161 = 0$$

and its approximate factors are:-

$$(\lambda^2 + 0.563\,\lambda + 16.6)(\lambda^2 + 0.0083\,\lambda + 0.00098) .$$

The biquadratics for the other values of M are similar in nature and split into factors in the same very sharp way.

The values of the periods and dampings are tabulated below. All the motions are stable.

M/

| M | Short oscillation | | Phugoid | |
|---|---|---|---|---|
| | Damping | Period (secs.) | Damping | Period (secs.) |
| 1.3 | 0.303 | 1.48 | 0.0051 | 174 |
| 1.5 | 0.281 | 1.54 | 0.0044 | 200 |
| 1.7 | 0.275 | 1.53 | 0.0039 | 227 |
| 1.9 | 0.273 | 1.52 | 0.0035 | 254 |

If a downwash factor, $\left(1 - \frac{\partial \varepsilon}{\partial \alpha}\right)$, of 0.5 had been taken instead of unity the rapid periods become 1.84, 1.97, 2.00 and 2.04 secs.

It is seen that there is a well-damped oscillation of rather short period (having in mind the size of the aeroplane) and a phugoid which is of nearly zero damping and very long period.

The large value of the coefficient of $\lambda^2$ together with the fact that $C w - X_u B$ splits the biquadratic very closely into the factors

$$\left[\lambda^2 - (Z_w + M_q)\lambda - U M_w\right]\left[\lambda^2 - X_u\lambda - \frac{g Z_u}{U}\right] = 0.$$

The two periods are therefore, very closely

$$\frac{2\pi}{\sqrt{-U M_w}} \quad \text{and} \quad 2\pi\sqrt{\frac{U}{-gZ_u}} \quad \left(= \sqrt{2}\cdot\frac{\pi U}{g}\right)$$

and the damping factors are approximately:-

$$-\frac{Z_w + M_q}{2} \quad \text{and} \quad -\frac{X_u}{2}.$$

A useful expression for the rapid period is $T = 2\pi K\sqrt{\frac{C_L}{g\,l\,dC_m/d\alpha}}$

where $K$ is radius of gyration, $l$ is the length used in the moment coefficient, and $\alpha$ is in radians.

It is evident that quite wide variations in the values of all the derivatives will not upset these conclusions, which really arise because the forward speed, $U$, is so large.

## Conclusions

The following conclusions should apply to most supersonic designs but, of course, as detailed calculations as possible should always be made, especially if it is suspected that any derivative is likely to behave abnormally.

The phugoid is so lightly damped and of such long period that it can presumably be neglected entirely in any study of controlled motion. The characteristics of the short period, which are important, can be easily found approximately. The period demands only a knowledge of $M_w$, the damping needs the values of $Z_w$ and $M_q$, of which $Z_w$ is easy to estimate fairly accurately, but $M_q$ not so easy (the present estimate is obviously very crude).

If/

If the same analysis is applied to a smaller design more like a guided weapon, the nature of the solution is not changed. The most significant numerical difference is that the rapid oscillation is then likely to have a surprisingly short period, often less than 0.5 sec. so that automatic controls might be found difficult to match to the response of the aircraft. This difficulty can be avoided either by making the period longer by reducing [illegible] to the lowest value judged to be safe, or by keeping the time lag in the servo control very short. This is a point which must always be closely watched, and the fact that the characteristics of the oscillation are so easy to calculate approximately, and the analytical expression of the motion merely a quadratic, is a great help and simplification in the study of controlled motion.

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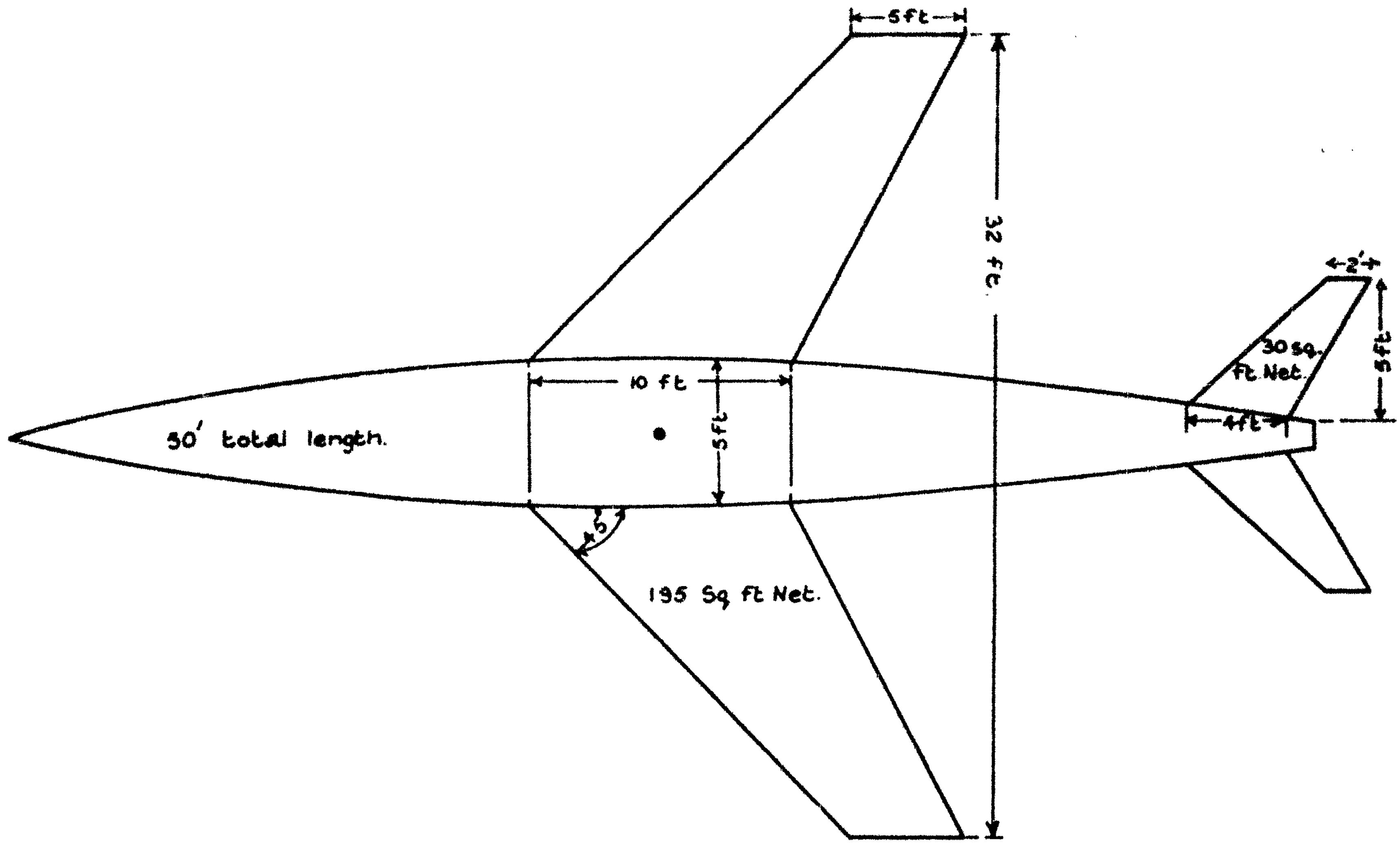
Fig. 1.




Assumed Outline of Aeroplane.

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